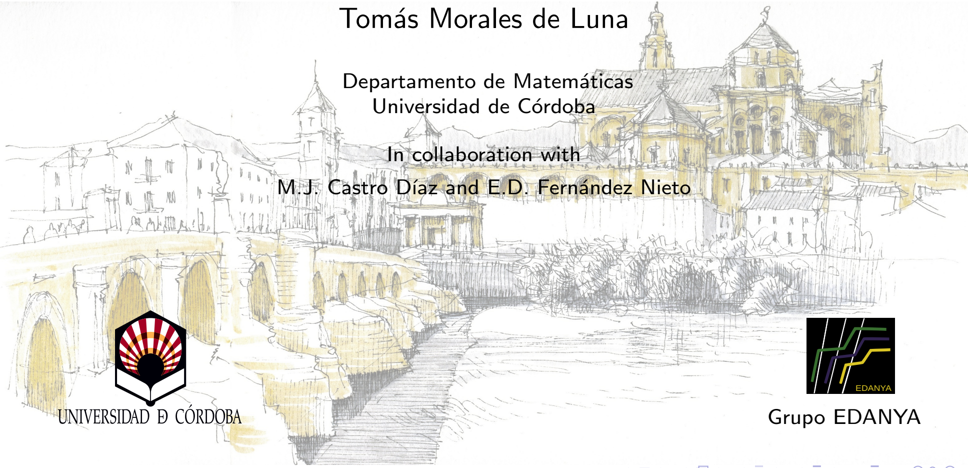
A multilayer approach for the simulation of suspended sediment transport



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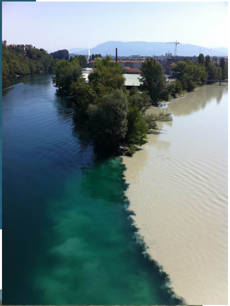
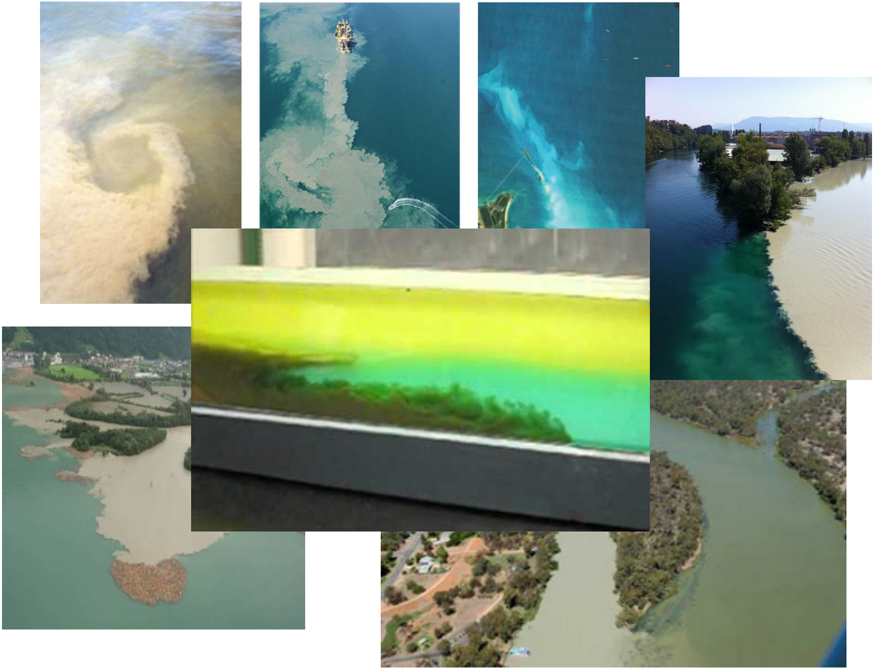
UNIVERSIDAD DE CÓRDOBA

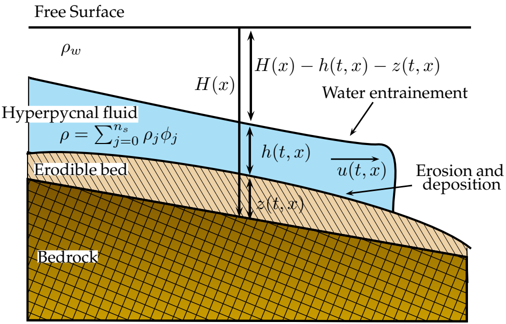
Grupo EDANYA

Outline

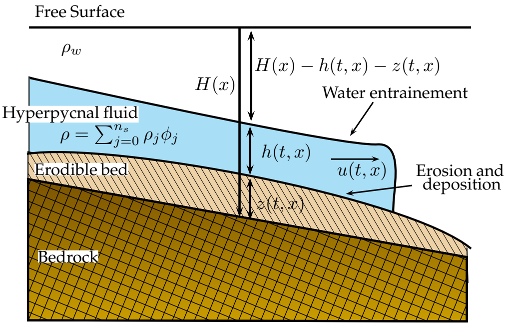






Free Surface
ρ_w
$H(x)$
$H(x) - h(t,x) - z(t,x)$
Water entrainement
Hyperpycnal fluid
$\rho = \sum_{j=0}^{n_s} \rho_j \phi_j$
$h(t,x)$
$u(t,x)$
Erodible bed
Erosion and deposition
$z(t,x)$
Bedrock



$$
\begin{cases}
\partial_t h + \partial_x(hu) = G_\eta + G_b, \\
\partial_t(hu) + \partial_x\left(hu^2 + g\left(R_0 + R_\phi\right)\frac{h^2}{2}\right) = \\
\qquad g\left(R_0 + R_\phi\right)h\partial_x(H - z) + uG_\eta + \frac{u}{2}G_b + \tau, \\
\partial_t(h\phi_j) + \partial_x(hu\,\phi_j) = G_b^j, \text{ for } j = 1, \dots, n_s \\
\partial_t z + \xi\partial_x q_b = -\xi G_b.
\end{cases}
$$

Bárcenas Gascón, P. (2012).
Procesos Morfogenéticos y Evolución Reciente de los Depósitos Prodeltaicos del Sureste de la Península Ibérica: Aplicaciones de Modelos Matemáticos.
PhD thesis, Universidad de Málaga.

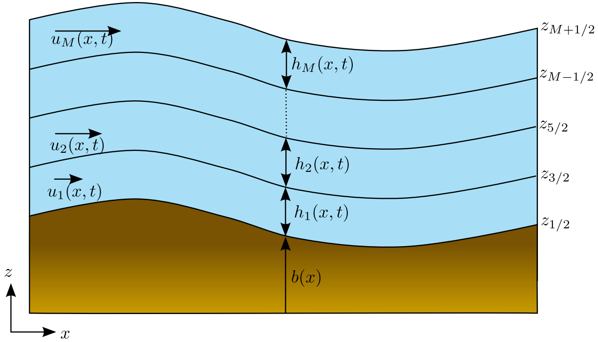
Vertical distribution of sediments?

Outline

1. A multilayer approach

2. Numerical simulations

A multilayer approach



N sediment species with density ρ_i and size d_i
ϕ_i = volumetric concentration $i = 1, \ldots, N$

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$$\phi = \sum_{i=1}^{N} \phi_i, \quad \phi_0 = 1 - \phi, \text{ and } \Phi = (\phi_0, \phi_1, \ldots, \phi_N)$$

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$$\phi = \sum_{i=1}^{N} \phi_i, \quad \phi_0 = 1 - \phi, \text{ and } \Phi = (\phi_0, \phi_1, \ldots, \phi_N)$$

$$\rho(\Phi) = \sum_{i=0}^{N} \rho_i \phi_i$$

$$v_i = (u_i, w_i), \quad i = 0, 1, \ldots, N,$$

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Average velocity

$$q = \sum_{i=0}^{N} \phi_i v_i$$

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Relative/slip velocity

$$\Delta v_i = v_i - v_0, \quad i = 1, \ldots, N$$

Local mass balance equations

$$\partial_t \phi_i + \nabla \cdot (\phi_i v_i) = 0, \quad i = 0, 1, \ldots, N,$$

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Sum of all equations:

$$\nabla \cdot q = 0$$

Momentum balance equations

$$\rho_i(\partial_t(\phi_i v_i) + \nabla \cdot (\phi_i v_i \otimes v_i)) = -\rho_i \phi_i g \vec{k} + \nabla \cdot \Sigma_j, \quad i = 0, 1, \ldots, N,$$

Momentum balance equations

$$\rho_i(\partial_t(\phi_i v_i) + \nabla\cdot(\phi_i v_i \otimes v_i)) = -\rho_i\phi_i g\vec{k} + \nabla\cdot\Sigma_j, \quad i = 0, 1, \ldots, N,$$

$\Sigma_i = -\phi_i pI + T_i^E$
p pressure
T_i^E viscous stress tensors

Denote by $q = (u, w)^t = \sum_{i=0}^{N} \phi_j v_j,$

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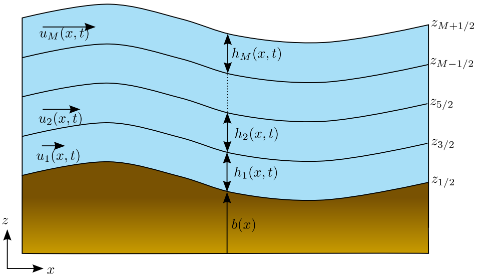
Hypothesis

$$\begin{aligned} u_j &= u \\ w_j &= w + \Delta w_j, \end{aligned} \qquad j = 1, \ldots, N.$$

A multilayer approach



- $M+1$ interfaces $\Gamma_{\alpha+\frac{1}{2}} = \left\{ z = z_{\alpha+\frac{1}{2}} \right\}$ for $\alpha = 0, \ldots, M$
- $\Omega_\alpha = \left\{ (x,z)/z_{\alpha-\frac{1}{2}} < z < z_{\alpha+\frac{1}{2}} \right\}$ for $\alpha = 1, \ldots, M$
- $\vec{n}_{\alpha+\frac{1}{2}}$ (space-time) outwards unit normal vector to the interface $\Gamma_{\alpha+\frac{1}{2}}$

Mass equation

$$\partial_t(\phi_{j,\alpha}h) + \partial_x(\phi_{j,\alpha}hu_\alpha) = \frac{1}{l_\alpha}\left(\frac{\phi_{\alpha+1}+\phi_\alpha}{2}G_{\alpha+\frac{1}{2}} - \frac{\phi_\alpha+\phi_{\alpha-1}}{2}G_{\alpha-\frac{1}{2}} - <\phi_j\Delta w_j>_{\alpha+\frac{1}{2}} + <\phi_j\Delta w_j>_{\alpha-\frac{1}{2}}\right)$$

Momentum conservation

$$
\begin{aligned}
&\partial_t(\rho(\Phi_\alpha)hu_\alpha) + \partial_x(\rho(\Phi_\alpha)hu_\alpha^2) + \partial_x\left(gh\left(\sum_{\alpha+1}^{M} l_\beta\rho(\Phi_\beta)h + \frac{g}{2}h\rho(\Phi_\alpha)\right)\right)\\
&= g\sum_{\beta=\alpha+1}^{M} l_\beta\rho(\Phi_\beta)h\partial_x h - g\rho(\Phi_\alpha)h\left(\partial_x b + \sum_{\beta=1}^{\alpha-1} l_\beta\partial_x h\right)\\
&\quad + \frac{1}{l_\alpha}\Bigg(\frac{u_{\alpha+1}+u_\alpha}{2}\,\frac{\rho(\Phi_{\alpha+1})+\rho(\Phi_\alpha)}{2}G_{\alpha+\frac{1}{2}}\\
&\quad - \frac{u_\alpha+u_{\alpha-1}}{2}\,\frac{\rho(\Phi_\alpha)+\rho(\Phi_{\alpha-1})}{2}G_{\alpha-\frac{1}{2}}\\
&- \frac{u_{\alpha+1}+u_\alpha}{2}\sum_{j=0}^{N}\rho_j < \phi_j\Delta w_j >_{\alpha+\frac{1}{2}} + \frac{u_\alpha+u_{\alpha-1}}{2}\sum_{j=0}^{N}\rho_j < \phi_j\Delta w_j >_{\alpha-\frac{1}{2}} + T_{j,\alpha}
\end{aligned}
$$

$$\partial_t \mathbf{w} + \partial_x \mathbf{F}(\mathbf{w}) + \mathbf{B}(\mathbf{w})\partial_x \mathbf{w} = \mathbf{S}(\mathbf{w})\partial_x b + \mathbf{G}(\mathbf{w}),$$

$$r_{j,\alpha} := \phi_{j,\alpha} h \text{ for } j = 0, 1, \ldots, N$$

$$q_\alpha := \rho(\Phi_\alpha) h u_\alpha,$$

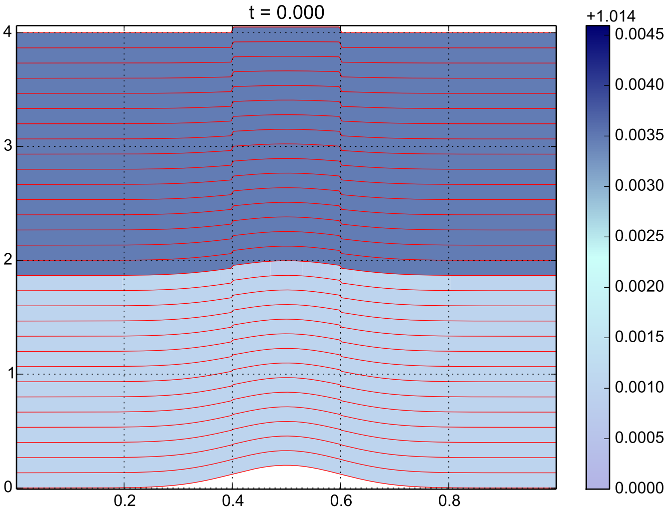
$$\mathbf{w} := (h, \mathbf{q}, \mathbf{r})^{\mathrm{T}} \in \mathbb{R}^{((N+1)M+1)}$$

$$m_\alpha = \rho_\alpha(\Phi)h.$$

$$\partial_t \widetilde{\mathbf{w}} \; + \; \partial_x \widetilde{\mathbf{F}}(\widetilde{\mathbf{w}}) + \widetilde{\mathbf{B}}(\widetilde{\mathbf{w}})\partial_x \widetilde{\mathbf{w}} \; = \; \widetilde{\mathbf{S}}(\widetilde{\mathbf{w}})\partial_x b + \widetilde{\mathbf{G}}(\widetilde{\mathbf{w}}),$$

$$\widetilde{\mathbf{w}} := (h, \mathbf{m}, \mathbf{q})^{\mathrm{T}} \in \mathbb{R}^{2M+1}$$

Outline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0.0040
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0.0030
0.0025
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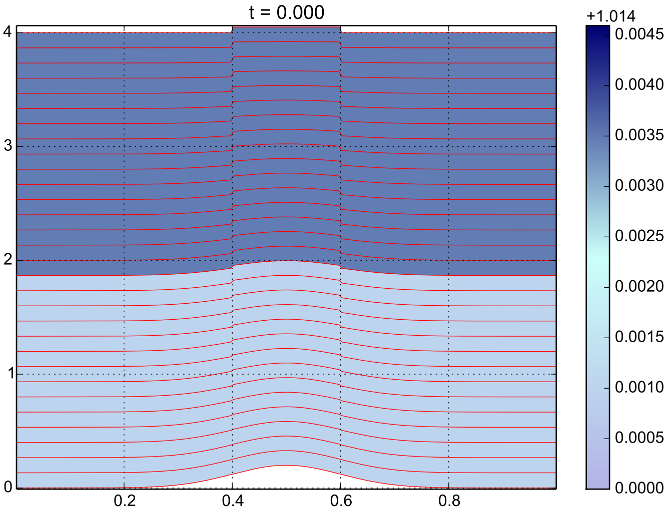
2
3
4
0.2
0.4
0.6
0.8


t = 0.000
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3
2
1
0
0.2
0.4
0.6
0.8
+1.014
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0.0040
0.0035
0.0030
0.0025
0.0020
0.0015
0.0010
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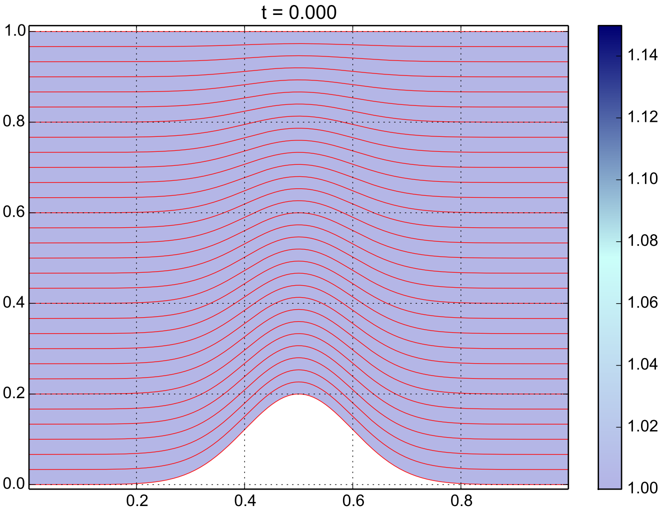
0.0000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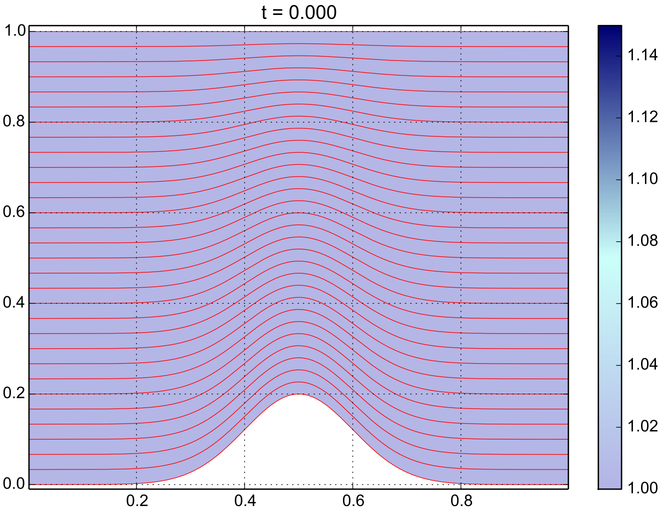
1.04
1.02
1.00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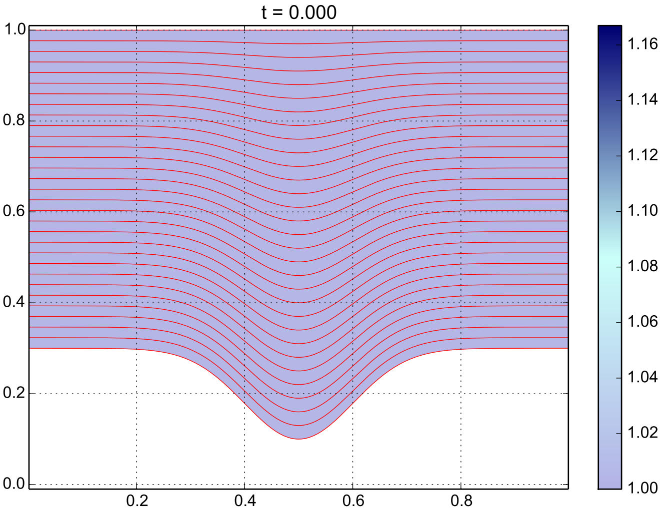
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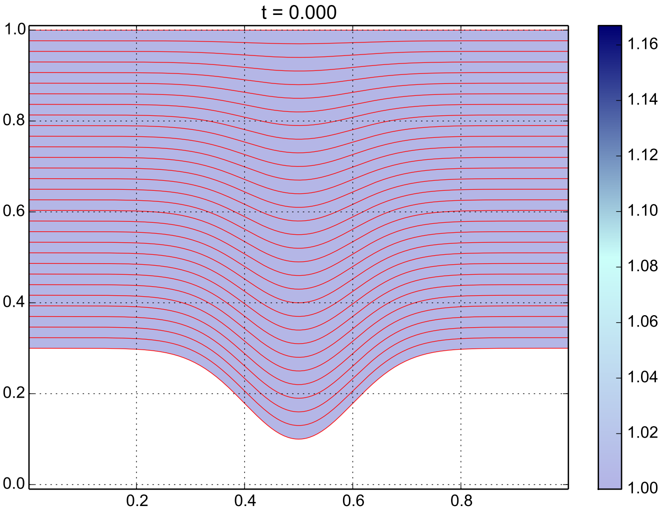
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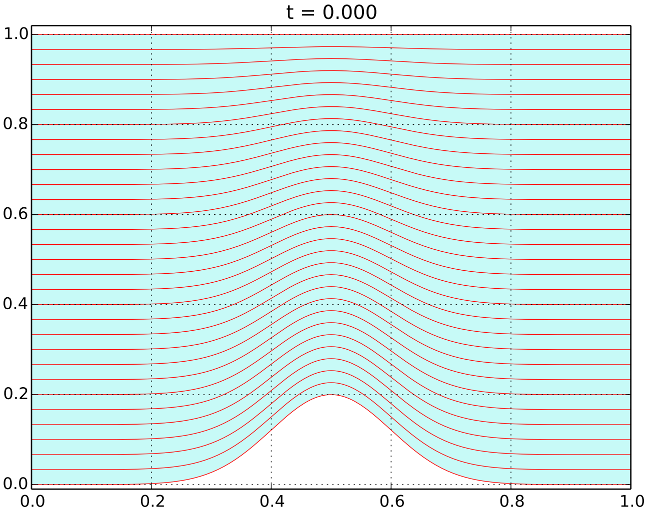
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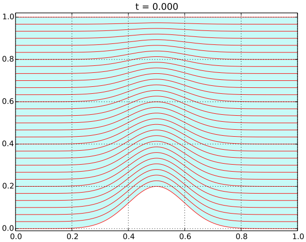
1.0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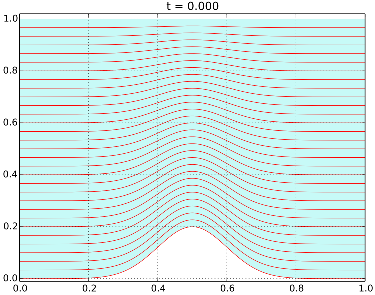
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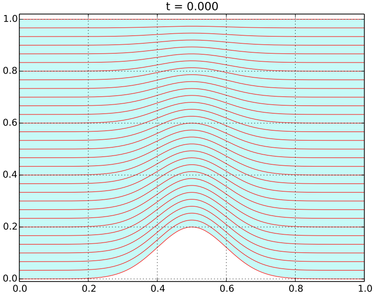
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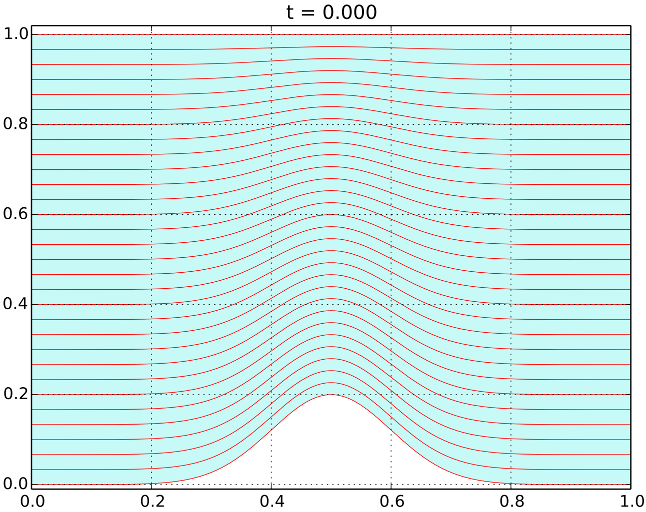
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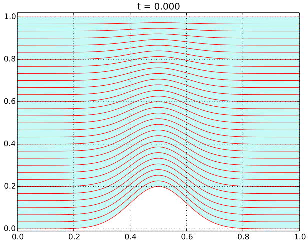
0.8
1.0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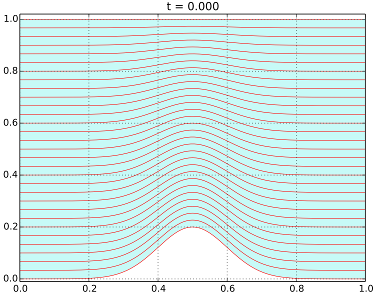
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0.8
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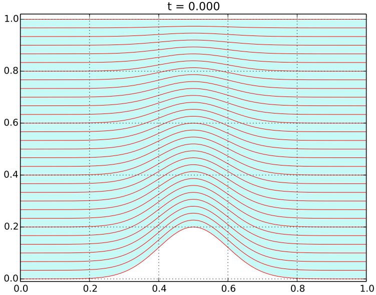
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0.2
0.4
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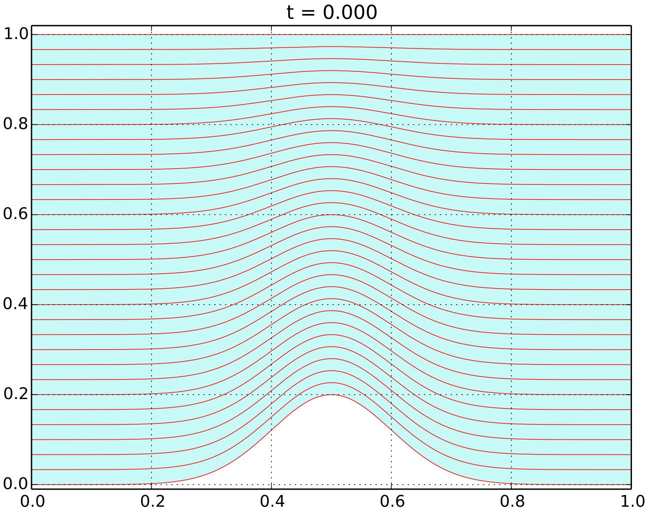
0.8
1.0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0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0

Thank you for your attention

Tomas.Morales@uco.es